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# IMPULSIVE MOTION OF SHEAR BUILDINGS INCLUDING PLASTICITY AND VISCOUS DAMPING

Edward Cohen,[1] J.M. ASCE, Leon S. Levy,[2] and Leonard E. Smollen,[3] J.M. ASCE

## SYNOPSIS

The method of normal modes is applied to the analysis of elasto-plastic shear buildings subjected to impulsive loads, foundation uplift and non-uniform viscous damping. Interaction of foundation rotation and plasticity in the superstructure is described. A short table of approximate modes and frequencies is included.

## INTRODUCTION

The use of the normal modes of vibration for the solution of dynamic problems involving shear buildings is extended to include the displacement of structures subjected to impulsive torsional loads, the development of plastic action in the frame and the foundation, "temporary instability" or uplift of the foundation and non-uniform viscous damping. The interaction of foundation rotation and plasticity in the superstructure is described and a general approach to the solution of the problem is outlined. Tables of modes and frequencies for typical buildings have also been included.

Shear buildings, as treated in this paper, are defined as idealized multistory structures with the following properties:

1. The mass of the building is concentrated at the various floor levels.
2. The motion of the building is due to shear only. The displacements due to flexure of the building as a whole are negligible.
3. The elastic shear in any story is a function only of the elastic deflection between adjacent floors.

For the purpose of dynamic analysis such idealized buildings are analogous to rigid frame structures with stiff floor systems and flexible columns and to many other structures of such height to width ratio that overall flexural deformation can be neglected. A typical five story shear building is shown in Fig. 1. The equations of motion for a multi-story shear building with impulsive lateral loads will be developed in terms of the normal modes of vibration. The use of normal modes is advantageous because the problem is thereby reduced from the solution of the N simultaneous equations that would be required by the direct application of Newton's equation of motion to that of N independent equations for a building with N degrees of freedom.

Numerical methods, such as finite difference, trial and error, etc. may be used effectively in the solution of the dynamic problems treated in this paper.

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However, the methods described are also applicable where it is possible to obtain analytic solutions. They have been used by the authors for the numerical analysis of a three story building subjected to a long duration blast loading. The computations were done by finite difference theory with the aid of only ordinary automatic desk type calculators. However, it is recognized that as the number of stories increases the volume of computations also increases rapidly and when the number of stories exceeds about five the use of high speed digital computers or analog computers becomes necessary.

### A. General Relationships

The equation of free vibration for the $i^{th}$ floor of a shear building (Fig. 2) on a rigid foundation is given by

$$M_i \ddot{x}_i = K_{i+1} (x_{i+1} - x_i) - K_i (x_i - x_{i-1}) \; (i = 1,2, \ldots n) \tag{1}$$

where

$M_i$ = The mass concentrated at the $i^{th}$ floor.

$K_i$ = Spring constant under the $i^{th}$ floor.

$x_i(t)$ = The absolute horizontal displacement of the $i^{th}$ floor at the time t.

The displacement at any time (t) may be expressed in terms of normalized modes $\phi_j$ and the generalized coordinates, $q_j(t)$, as

$$x_i(t) = \sum_{j=1}^{N} \phi_j(i) \, q_j(t) \tag{2}$$

By application of the Lagrangian equations,

$$\frac{d}{dt} \frac{\partial T}{\partial q_k} + \frac{\partial U}{\partial q_k} = Q_k \tag{3}$$

where:

$$Q_k = \sum_{i=1}^{N} P_i \frac{\partial x_i}{\partial q_k} = \text{the generalized force} \tag{4}$$

$P_i$ = the real force applied to the $i^{th}$ floor

$$T = \text{the kinetic energy} = \frac{1}{2} \sum_{i=1}^{N} \dot{q}_j^2 \tag{5}$$

$$U = \text{the potential energy} = \frac{1}{2} \sum_{i=1}^{N} \omega_j^2 q_j \tag{6}$$

N = the number of stories

$\omega_j$ = the frequence of the $j^{th}$ mode

The equations of forced vibration in terms of the generalized coordinates become

$$\ddot{q}_j + \omega_j^2 q_j = \sum_{i=1}^{N} P_i \phi_j(i) \quad (j = 1,2, \ldots N) \tag{7}$$

Thus far it has been assumed that the story masses undergo horizontal motion only. This assumption is valid when the foundation material is infinitely rigid. When the foundation is elastic, the building will also rotate about an axis through the centroid of the base and perpendicular to the direction of motion as shown in Fig. 3.

For this case, the following equation of motion may be obtained:

$$\ddot{q}_k + \Omega_k^2(t) = \sum_{i=1}^{N} P_i \Psi_k(i) \quad (k = 1,2,\ldots N = 1) \tag{8}$$

where $q_k$, $\Omega_k$, and $\Psi_k(i)$ are the modified generalized coordinates, frequencies, and modes of a building rotating elastically about the centroid of its base.[4]

$\Psi_k(i)$ may be expressed as

$$\Psi_k(i) = C_k \phi_A(i) + \sum_{j=1}^{N} C_{kj} \phi_j(i); \quad (k = 1,2,\ldots, N+1) \tag{9}$$

where the coefficients $C_k$ and $C_{kj}$ are given by

$$\frac{\omega_A^2}{\Omega_k^2} C_k = \sum_{i=1}^{N} \phi_A(i) \Psi_k(i) M_i \tag{10}$$

and

$$\frac{\omega_A^2}{\Omega_k^2} C_{kj} = \sum_{i=1}^{N} \phi_j(i) \Psi_k(i) M_i \tag{11}$$

$$\phi_A(i) = \frac{Li}{\sqrt{I_0}} = \text{the ``rocking mode'' of the building} \tag{12}$$

$I_0$ = the total moment of inertia about the axis of rotation

The rotation may be expressed as

$$\theta = \sum_{k=1}^{N+1} \overline{\theta}_k q_k \tag{13}$$

where

$$\overline{\theta}_k = C_k \overline{\theta} = C_k \frac{\phi_A}{L_i} = \frac{C_k}{\sqrt{I_0}} \tag{14}$$

The above equations apply to the case of structures subjected to impulsive loads such as may be caused by blast or similar phenomena. The case of forced vibrations caused by impressed displacements has been treated previously.[5]

4. M.G. Salvadori. "Earthquake Stresses in Shear Buildings," Proceedings, A.S.C.E., March, 1953, vol. 79, Separate No. 177.
5. op. cit.

B. Structure Subjected to Impulsive Torque Loading

When a structure supported on an elastic foundation is subjected to vertical impulsive loads having a moment, $M_R$, (see Fig. 4) about the centroid of the base, an additional term must be added to Equation (8). By considering the work done by the moment, an additional generalized force due to the moment is

$$Q_k = \frac{\partial W}{\partial q_k} = M_R C_k \frac{\phi_A(i)}{L_i} = M_R C_k \bar{\theta} = M_R \frac{C_k}{\sqrt{I_0}} \tag{15}$$

and the equation of motion in terms of the generalized coordinates becomes

$$\ddot{q}_k + \Omega_k^2 = \sum_{i=1}^{N} P_i \Psi_k(i) + MC_k\bar{\theta} \tag{16}$$

When the moment is caused by vertical loads, and the rotations are small, the effect of the equivalent direct force through the centroid and that of the couple may be computed independently in the elastic range or as described in Section D, when the uplift or plastic action occurs in the foundation.

C. Plastic Hinges Between Stories in Building on Rigid Foundation

Plastic hinges are said to develop between any two floors, say the $i^{th}$ and the $(i+1)^{th}$, when the relative displacement of the $i^{th}$ and $(i+1)^{th}$ floor masses are such that the shear, $K_{i+1}(x_{i+1} - x_i)$ exceeds the yield point resistance. (Fig. 5 shows an idealized plot of shear resistance versus displacement). From the time a plastic hinge first develops until such time as the relative velocities of the two adjacent floors reverse and the story shear returns to the elastic range, the method of analysis must take into account the first, and all subsequent, plastic hinges as they develop.

If the problem is solved analytically, it is necessary to revise the equations of motion whenever any story exceeds its yield point displacement. This requirement also holds for a numerical solution. Since the duration of the time intervals must be kept to a small fraction[6] of the period of the given mode being computed, the possibility that a new hinge may develop within any one step is usually unimportant, the change in equations being made either at the beginning or the end of that time interval.

When considering a shear building in which plastic hinges have developed between certain stories, it is clear that the plastic hinges divide the building into a number of smaller "independent" structures or substructures, as shown in Fig. 6, each with its own modes and frequencies of vibration. Therefore, the shear building analysis can be applied to each of the "substructures" once certain modifications due to the plastic resistance, $S_p(i)$, have been applied and the initial conditions established. (Relative motion between the foundation and the soil may be treated in exactly the same way.)

It should be noted that for a building on a rigid foundation the development of a plastic hinge between any two floors reduces the problem to the solution of two independent "substructures." Thus, a four story building in which the

6. The maximum duration of the time interval relative to the modal period is a function of the numerical method used and the required accuracy. It may also be modified by the variation with time of the loading.

third story has become plastic is reduced to two independent 2-story buildings with a consequent simplification of the problem and in the case of numerical solutions, a great reduction in the required labor.

If r stories are isolated by plastic hinges they may be considered to form an r-story structure. The equation of motion of the new r-story structure are also given by the Lagrange equations,

$$\frac{d}{dt}\frac{\partial T}{\partial q_k} - \frac{\partial T}{\partial q_k} = -\frac{\partial \overline{U}}{\partial q_k} + Q_k \quad (k = 1,2, \dots r) \tag{17}$$

where the kinetic energy T, and the potential energy U, are for the new system now under consideration.

The kinetic energy is defined as before, i.e.;

$$T = \frac{1}{2}\sum_{i=1}^{r} M_i \dot{x}_i^2 = \frac{1}{2}\sum_{i=1}^{r} \dot{q}_j^2 \tag{18}$$

The potential energy is defined to include the effect of both the internal and external work done. This will also include the work done in overcoming the plastic resistances, $S_p(i)$, so that once plastic hinges have developed, the potential energy, $\overline{U}$ is equal to U, as previously defined, plus the work done by the plastic resistances.

$$\overline{U} = U + S_{p(c+r)}\, x_{(c+r)} - S_{p(c+1)}\, x_{(c+1)} \tag{19}$$

or

$$\overline{U} = U + S_{p(c+r)} \sum_{j=1}^{r} \phi_j(r) q_j - S_{p(c+1)} \sum_{j=1}^{r} \phi_j(i)\, q_j \tag{20}$$

The displacements, $x_i$, can be computed from:

$$x_i = \sum_{j=1}^{r} \phi_j(i)\, q_j$$

The initial conditions for the generalized coordinates of any substructure isolated by the development of a plastic hinge are determined from the value of $x_i^p$ and $\dot{x}_i^p$ at the instant the plastic hinge is developed.

At the moment plasticity occurs:

$$x_i^p = \sum_{j=1}^{r} \phi_j(i)\, q_j(t) \tag{21}$$

where the superscript p defines that instant and $\phi_j$ are the modes of the r-story "substructures."

Multiplying both sides by $\phi_j(i)\, M_i$ and summing over i, we obtain:

$$\sum_{i=1}^{r} \phi_j(i)\, M_i x_i^p = \sum_{j=1}^{r} \sum_{i=1}^{r} \phi_k(i)\, \phi_j(i)\, M_i\, q_j^p \tag{22}$$

However,

$$\sum_{j=1}^{r} \sum_{i=1}^{r} \phi_j(i)\, \phi_k(i)\, M_i = 1 \tag{23}$$

and

$$q_j^p = \sum_{i=1}^{r} \phi_j(i)\, M_i\, x_i^p \tag{24}$$

Similarly,

$$\dot{q}_j^p = \sum_{i=1}^{r} \phi_j(i)\, M_i\, \dot{x}_i^p \tag{25}$$

### D. Temporary Instability and Plasticity in the Foundation Soil

The analysis presented for a rocking building is valid only when the building can be considered as rotating about the centroid of the foundation. This is approximately true only when the building exerts a compressive force along the entire base. However, when the analysis indicates tension on part of the foundation the building will pull actually away from the ground, and the center of rotation will shift.

From the soil pressure analysis, a force diagram such as shown in Fig. 7 may be found.

The fictitious tensile forces which must be eliminated are equivalent to a vertical downward force, T, through the centroid of the base, and a counterclockwise moment, TE, where E is the lever arm of the centroid. An upward vertical force, T', and a clockwise moment, T'E, equal in magnitude to the quantities to be subtracted are therefore applied to the structure.

Using a step-by-step finite difference method, a solution by successive trials may be obtained as follows:

1. Assume a value for the vertical force T and the moment TE.
2. Compute the displacements $x_i$ from the equations of Section B by adjusting Equation (16) so that:

$$\ddot{q}_k + \Omega_k^2 q_k = \sum_{i=1}^{N} P_i \psi_k(i) + M_R\, c_k \frac{\phi_{A(i)}}{L_i} + \lambda_k \tag{26}$$

where

$$\lambda_k = \frac{(T'E)\, c_k\, \phi_A(i)}{L_i} \tag{27}$$

3. Determine the soil pressure diagram based on the values assumed and compute T and TE.
4. If a check is obtained, proceed to the next time interval. If not, repeat Steps 1 to 3.

It may be noted that the proposed method of solution results in a uniformly asymptotic approach to the final condition of equilibrium, the computed resisting moment of the foundation being always greater and the rotation always smaller than the actual values.

Plastic action in the soil may be accounted for in a similar manner by equilibrating the difference between the plastic and elastic soil resistance, as shown in Fig. 8.

### E. Rotation of Building with Large Plastic Distortions in Superstructure

Plastic hinges in a building rotating about the center of its base divide it into a number of independent substructures, any one of which may be treated separately, once the effects of the plastic hinges above and below are included

in the analysis. Each plastic hinge is assumed to transmit only a horizontal plastic resistance, $S_{pi}$, an axial force, $T_i$, and a moment, $M_i$. Throughout this section it will be assumed that the plastic resistance, $S_{pi}$, is known at any given time, t, that it acts in a horizontal direction and may be treated as an additional applied force.

If it is also assumed that the plastic story is axially stiff, i.e., the column lengths do not change, both the columns in each story and the floor masses in each substructure remain parallel and the rotations of all stories are identical. The distorted plastic story is then equivalent to a pin connected parallelogram, or a series of parallelograms when the number of columns exceeds two. The adjacent masses move both horizontally and vertically relative to each other as the structure rotates. The new unknowns are the horizontal and vertical components of the column reactions and the story to story distortion, $\alpha$, as modified by rotation.

Fig. 9(a) shows the plastic hinge at time, t, where $\overline{AB}$ is the relative displacement of the upper substructure at the plastic hinge, and $\alpha$ is the angle of distortion of the plastic story. Fig. 9(b) shows the resultant, T, of the final axial column forces, and their couple, M, resulting from the rotation. This moment is a measure of the contribution of the upper "substructure" to the total rotational inertia of the structure. The contribution diminishes as the distortion, $\alpha$, increases. The resultant force, T, which is superimposed on the pure couple is caused by applied vertical loads and the vertical inertia forces. The vertical and horizontal components of the plastic displacements, $\overline{AB}$, are shown in Fig. 10.

The following parameters are then obtained for an N-story building with n plastic hinges:

N shear displacements $x_i$

n+ 1 rotations $\theta_i$

n plastic axial forces $T_i$

n plastic displacement angles $\alpha_i$

We also have the following independent equations:

N equations for shear displacement

n+1 equations for rotation

n equations of rotational compatibility, i.e., $\theta_i = \theta_{i+n}$

n equations of displacement compatibility

The initial conditions (at rest) are

$$x_i^o = 0 \qquad \dot{x}_i^o = 0$$

$$\theta_i^o = 0 \qquad \dot{\theta}_i^o = 0$$

$$\alpha_i^o = 0 \qquad \dot{\alpha}_i^o = 0$$

$$T_i^o = 0 \qquad \dot{T}_i^o = 0$$

For plastic hinge development at any later time, t, suitable initial conditions may be derived, by the procedure outlined in Section C.

For a step-by-step solution where the number of plastic hinges is small (less than 3 or 4), $x_i$ and $\theta_i$ at the end of any time interval may be found by

trial and error by assuming values for $\alpha_i$ and $T_i$ and computing $\theta_i$ and $x_i$. The equations of shear displacement and displacement compatibility serve as a check on the accuracy of the assumed values of $\alpha_i$ and $T_i$. This method is restricted to a structure with a limited number of hinges because the solution itself has no inherent properties of convergence and depends entirely on the computer's judgment and ability to visualize the action of the structure.

## F. Unequal Viscous Damping in Complementary System

Due to the large difference in magnitude which may exist between the damping due to the soil foundation and that due to the shear displacements of the structure, it is desirable to account for both these factors, i.e., evaluate the actual damping coefficients, $\mu_k$, of the $k^{th}$ mode of the rotating structure in terms of the damping coefficients $\mu_A$ of the foundation and $\mu_j$ of the shear modes of the structure on a rigid foundation. Both $\mu_j$ and $\mu_A$ may be determined experimentally.

It may be noted that in Equation (1), the equation of motion of any given floor mass, viscous damping may be introduced by the addition of linear velocity terms, so that Equation (1) becomes:

$$M_i\ddot{x}_i = K_{i+1}(x_{i+1} - x_i) - K_i(x_i - x_{i+1}) + \sum_{I=1}^{N} \mu_{iI}(\dot{x}_I) \tag{28}$$

where $\mu_{iI}$ is the coefficient of the damping between the $I^{th}$ and $i^{th}$ floor masses and where, in general, only the $I = (i+1)$, i, and (i-1) velocities affect the motion of the $i^{th}$ floor. Therefore

$$M_i\ddot{x}_i = K_{i+1}(x_{i+1} - x_i) - K_i(x_i - x_{i-1}) + \mu_{i(i+1)}(\dot{x}_{i+1} - \dot{x}_i) - \mu_{i(i-1)}(\dot{x}_i - \dot{x}_{i-1}) \tag{29}$$

The damping coefficients which account for the effect of one story mass upon another are difficult to evaluate directly even in the case where rotation is not allowed.

However, if it is assumed that the damping of single degree of freedom systems is a linear function of the velocity, a damping term may be added to the equation for the generalized coordinates, which becomes:

$$\ddot{q}_k + 2\mu_k\dot{q}_k + \Omega_k^2 q_k = \sum_{i=1}^{N} p_i\psi_k(i) + M_R c_k \overline{\theta} \tag{30}$$

The damping coefficient, $\mu_k$, may be evaluated in terms of $\mu_j$ and $\mu_A$ by considering the dissipative function of the damping.

A dissipative function,[7] D, can be defined such that the damping force is the negative of the derivative of the dissipative function with respect to the velocity. The dissipative function may also be defined by

$$D = -\frac{1}{2} W \tag{31}$$

where W is the instantaneous work, so that -W is the time rate of energy dissipation. The dissipative function may be introduced into the Lagrange equations, so that:

7. Karman and Biot, "Mathematical Methods in Engineering," McGraw-Hill, 1940, p. 219.

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} + \frac{\partial D}{\partial \dot{q}_k} = 0 \tag{32}$$

Because of the second definition of the dissipative function, the total dissipative function may be equated to the sum of the soil and shear dissipative functions.

$$D_V = D_{Vs} + D_{V\theta} \tag{33}$$

where $D_V$, the total dissipative function, is:

$$D_V = \sum_{k=1}^{N+1} \mu_k \dot{q}_k^2 \tag{34}$$

$D_{Vs}$, the shear dissipative function, is:

$$D_{Vs} = \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \dot{x}_{is}\dot{x}_{Is} = \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \sum_{k=1}^{N} \Psi_{ks}(i)\, \dot{q}_k \sum_{k=1}^{N+1} \Psi_{ks}(I)\dot{q}_k \tag{35}$$

and $D_{V\theta}$, the soil dissipative function, is:

$$D_{V\theta} = \mu_A \sum_{k=1}^{N+1} \theta_k^2 \dot{q}_k^2 \quad \text{where } \overline{\theta}_k = \frac{C_k}{\sqrt{I}} = C_k \frac{\phi_A(i)}{L_i} \tag{36}$$

Substituting in the above equation the values of the dissipative function, and differentiating with respect to the $k^{th}$ generalized velocity, we obtain:

$$\mu_k = \mu_A \overline{\theta}^2 + \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \Psi_{ks}(i)\, \Psi_{ks}(I) \tag{37}$$

If we now proceed to the limit where $\overline{\theta}_k$ 0 then:

$$\mu_j = \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \phi_j(i)\, \phi_j(I) \tag{38}$$

Noting that $\mu_{iI} = \mu_{Ii}$, $\mu_{ii} = \mu_{i(i+1)} + \mu_{i(i-1)}$ and that the $\mu_{iI}$, where I represents any story other than the $(i+1)^{th}$, $i^{th}$, or $(i-1)^{th}$, is zero, and knowing $\mu_j$ Equation (38) may be solved for the pertinent $\mu_{iI}$ in terms of $\mu_j$. These may then be substituted in Equation (37) to obtain the required $\mu_k$.

Note: Equation (35) has been used since it is a general mathematical expression for a dissipative function in a building of N stories. In physical terms it represents the damping action between any two (not necessarily adjacent) stories, i and I,. The damping for any floor mass as given by the equation for D substituted in the Lagrange equation is equal to $\partial D/\partial \dot{q}_k$ so that the damping for the $5^{th}$ floor mass, for example, will be given by the following expression:

$$\frac{\partial D}{\partial \dot{x}_5} = \frac{\partial}{\partial \dot{x}_5}\left[ \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \dot{x}_{is}\dot{x}_{Is} \right] = \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \frac{\partial \dot{x}_{Is}}{\partial \dot{x}_5} \dot{x}_{is}$$

$$+ \sum_{i=1}^{N} \sum_{I=1}^{N} \mu_{iI} \dot{x}_{is} \frac{\partial \dot{x}_{Is}}{\partial \dot{x}_5}$$

If the effect of damping in the shear modes is small enough to be neglected relative to the soil damping, Equation 37 becomes

$$\mu_k = \mu_A \, \bar{\theta}_k^2 \tag{39}$$

## G. Tables of Modes and Frequencies

Approximate modes and frequencies of some typical shear buildings having 2 to 5 stories have been tabulated in Table 1.

Notation

$M_i$ = mass of the $i^{\underline{th}}$ story

$K_i$ = stiffness of the $i^{\underline{th}}$ story

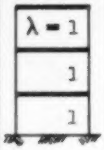
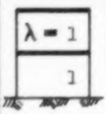
$$\beta = \frac{K_1}{M_1}$$

$$\lambda = \frac{K_i}{K_1}$$

| Building | i | $\frac{M_i}{M_1}$ | $\phi_1\sqrt{M_1}$ | $\phi_2\sqrt{M_1}$ | $\phi_3\sqrt{M_1}$ | $\phi_4\sqrt{M_1}$ | $\phi_5\sqrt{M_1}$ |
|---|---|---|---|---|---|---|---|
| $\lambda = 1$ | 5 | 0.75 | 0.615 | -0.599 | 0.555 | -0.456 | 0.278 |
| 1 | 4 | 1.00 | 0.575 | -0.265 | -0.205 | 0.559 | -0.500 |
| 1 | 3 | 1.00 | 0.483 | 0.268 | -0.590 | -0.067 | 0.587 |
| 1 | 2 | 1.00 | 0.348 | 0.599 | 0.104 | -0.496 | -0.512 |
| 1 | 1 | 1.00 | 0.182 | 0.480 | 0.608 | 0.531 | 0.297 |
| $\lambda = 0.2$ | 5 | 0.40 | 0.837 | -1.03 | 0.835 | -0.212 | 0.028 |
| 0.4 | 4 | 0.80 | 0.696 | -0.116 | -0.710 | 0.485 | -0.115 |
| 0.6 | 3 | 0.80 | 0.506 | 0.443 | -0.170 | -0.776 | 0.407 |
| 0.8 | 2 | 1.00 | 0.322 | 0.552 | 0.399 | 0.098 | -0.650 |
| 1.0 | 1 | 1.00 | 0.150 | 0.326 | 0.365 | 0.551 | 0.660 |
| $\lambda = 1$ | 5 | 1.00 | 0.599 | -0.546 | 0.462 | -0.322 | 0.165 |
| 1 | 4 | 1.00 | 0.550 | -0.170 | -0.333 | 0.595 | -0.456 |
| 1 | 3 | 1.00 | 0.456 | 0.326 | -0.550 | -0.165 | 0.604 |
| 1 | 2 | 1.00 | 0.326 | 0.599 | 0.174 | -0.452 | -0.555 |
| 1 | 1 | 1.00 | 0.170 | 0.456 | 0.596 | 0.546 | 0.331 |
| $\lambda = 1$ | 4 | 1.00 | 0.658 | -0.582 | 0.434 | -0.233 | |
| 1 | 3 | 1.00 | 0.577 | 0.00 | -0.582 | 0.582 | |
| 1 | 2 | 1.00 | 0.429 | 0.582 | -0.224 | -0.658 | |
| 1 | 1 | 1.00 | 0.228 | 0.582 | 0.658 | 0.429 | |

| Mode | $\omega_j/\sqrt{\beta}$ |
|---|---|
| 5 | 1.93 |
| 4 | 1.72 |
| 3 | 1.35 |
| 2 | 0.867 |
| 1 | 0.298 |
| 5 | 1.61 |
| 4 | 1.29 |
| 3 | 0.962 |
| 2 | 0.668 |
| 1 | 0.290 |
| 5 | 1.92 |
| 4 | 1.68 |
| 3 | 1.30 |
| 2 | 0.831 |
| 1 | 0.287 |
| 4 | 1.88 |
| 3 | 1.53 |
| 2 | 1.00 |
| 1 | 0.346 |

Table I

| Building | $i$ | $\frac{M_i}{M_1}$ | $\phi_1\sqrt{M_1}$ | $\phi_2\sqrt{M_1}$ | $\phi_3\sqrt{M_1}$ | $\phi_4\sqrt{M_1}$ | $\phi_5\sqrt{M_1}$ |
|---|---|---|---|---|---|---|---|
|  | 3 | 1.00 | 0.736 | -0.586 | 0.328 | | |
| | 2 | 1.00 | 0.591 | 0.327 | -0.736 | | |
| | 1 | 1.00 | 0.327 | 0.743 | 0.591 | | |
|  | 2 | 1.00 | 0.854 | -0.528 | | | |
| | 1 | 1.00 | 0.528 | 0.854 | | | |

| Mode | $\omega_j/\sqrt{\beta}$ |
|---|---|
| 3 | 1.80 |
| 2 | 1.24 |
| 1 | 0.445 |
| 2 | 1.62 |
| 1 | 0.616 |

Table I

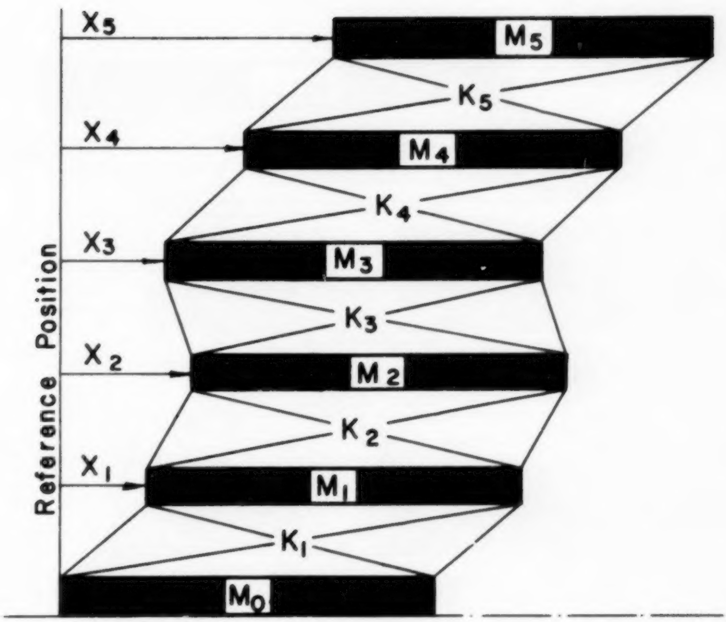



Fig. I DISTORTED POSITION OF IDEALIZED FIVE STORY BUILDING ON A RIGID FOUNDATION

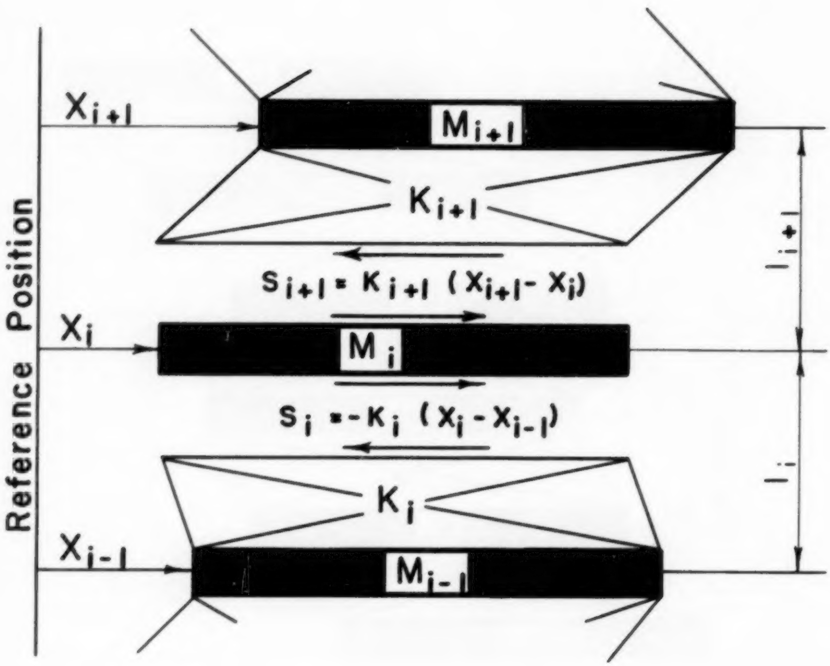



Fig. 2 TYPICAL FORCE SYSTEM FOR $i^{TH}$ FLOOR

$$X_i = L_i\theta + X_{i,s}$$

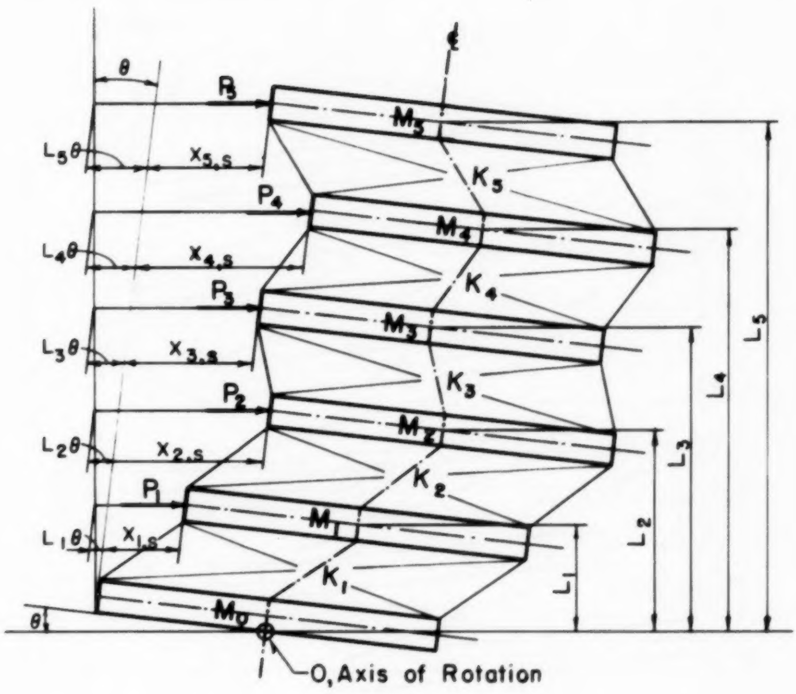



Fig. 3 DISTORTED POSITION OF IDEALIZED BUILDING ON A NON-RIGID FOUNDATION

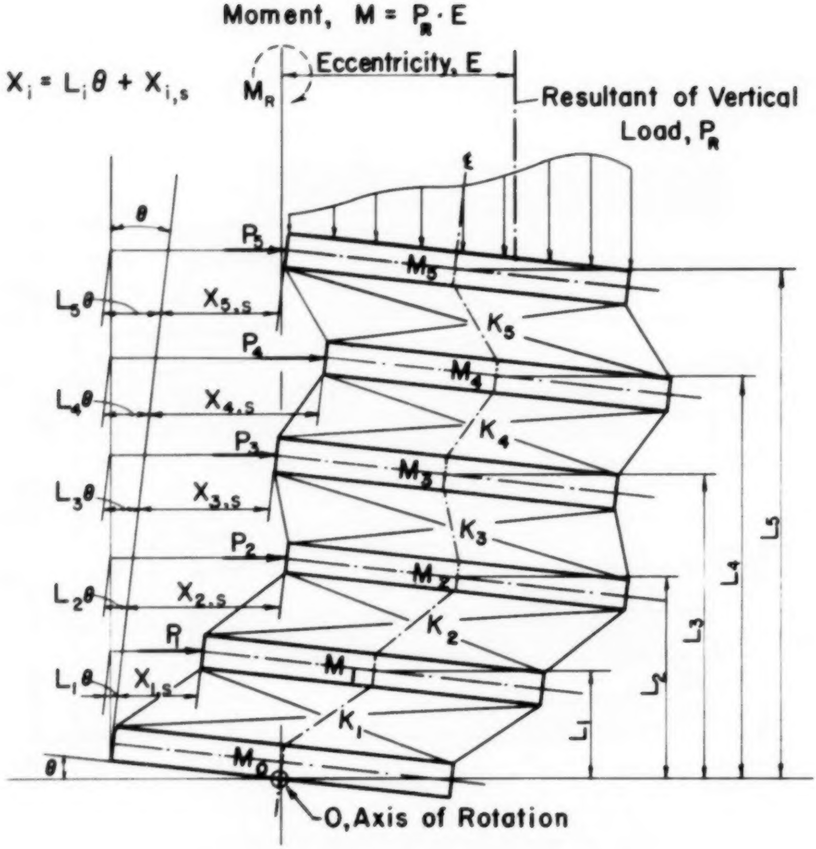



Fig. 4 DISTORTED POSITION OF IDEALIZED BUILDING ON AN ELASTIC FOUNDATION

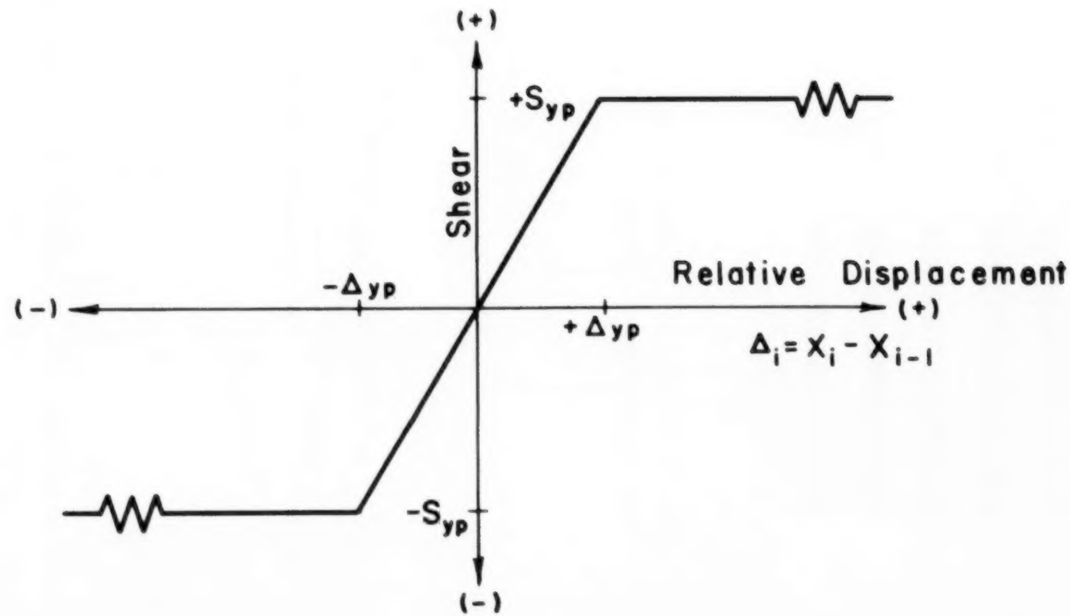



Fig. 5 IDEALIZED CURVE OF SHEAR RESISTANCE vs. DISPLACEMENT

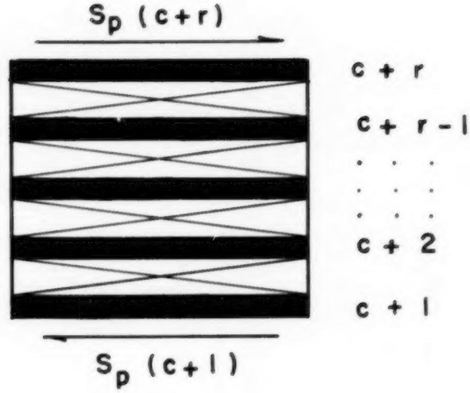



Fig. 6 PLASTIC FORCES ON "SUBSTRUCTURES"

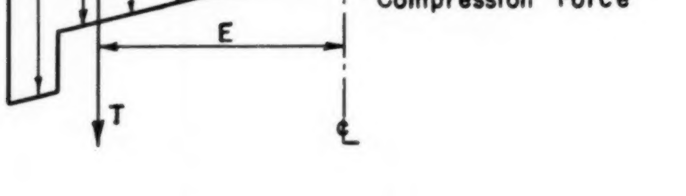



Fig. 7 FORCE DIAGRAM ON FOUNDATION FOOTING

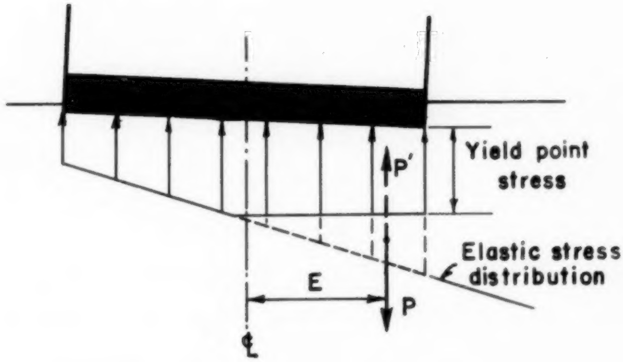



Fig. 8 PLASTIC ACTION IN SOIL

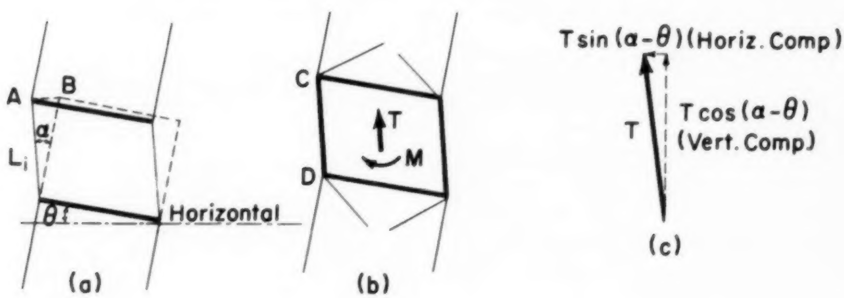



Fig. 9 REACTIONS AT PLASTIC HINGE

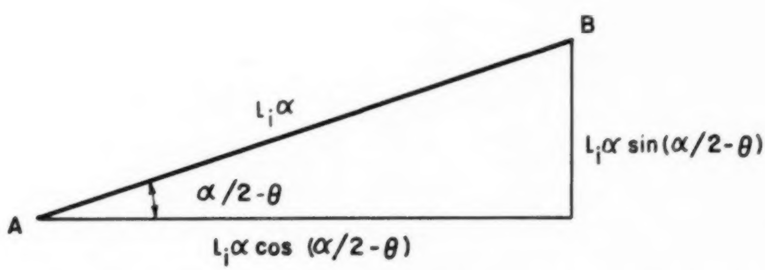



Fig. 10 VERTICAL AND HORIZONTAL COMPONENTS OF PLASTIC DISPLACEMENT

## PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

### VOLUME 80 (1954)

APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

### VOLUME 81 (1955)

JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

c. Discussion of several papers, grouped by Divisions.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.